

Evaluation of the Security of ESIGN Identification

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January 11, 2001

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Chapter 1

Introduction

The goal of this document is to evaluate the security of the identification scheme ESIGN Identification.

The security of ESIGN Identification depends on the security of the ESIGN signature scheme which, in turn, relies on the intractability of factoring integers and on the security of the used hash function and pseudorandom number generator. In order to evaluate the security of ESIGN Identification, it is, therefore, necessary to evaluate the difficulty of factoring integers and the security of the used hash function and pseudorandom number generator. But this is not sufficient. Even if the underlying number theoretic problem is hard and the hash function and random number generator are secure, ESIGN may still be insecure. A famous example for such a situation is the discovery of the possibility of an attack against the RSA encryption standard PKCS #1 (see [3]). In this standard, an insecure padding scheme was used, which compromised the security of the whole scheme.

Because of this situation, it is desirable to find a security proof for ESIGN Identification. Unfortunately, no provably hard computational problem in number theory is known, which could serve as the basis of a secure signature scheme. Also, no provably secure cryptographic hash function and pseudorandom number generator are known. Therefore, given current knowledge, there are no provably secure identification schemes. But it is possible to say more about the security of an identification scheme than just arguing that the underlying computational problems are intractable. Modern security proofs for digital signature schemes, on which many identification schemes including

ESIGN Identification are based, reduce their security to the difficulty of basic computational problems in mathematics. This means that the difficulty of those basic problems is not only necessary but also sufficient for the security of the digital signature scheme which relies on their security.

The questions that I answer in this report are the following. Which are the basic computational problems, on which the security of ESIGN Identification is based, and to what extent can this security be reduced to the intractability of those problems? How difficult are those basic mathematical problems and how difficult are the instances which arise from the specific applications in ESIGN Identification?

This report is organized as follows. Chapter 2 gives an overview over ESIGN and ESIGN Identification. Chapter 3 describes the models and techniques that are used in the security proofs of digital signature schemes and explains to what extent ESIGN can be proved secure in those models. The security of ESIGN relies on the intractability of factoring integers, finding collisions of hash functions and guessing the output of a pseudorandom number generator. Chapter 4 describes the current knowledge concerning the intractability of those basic problems. Chapter 5 describes the specific instances of the basic computational problems on which the security of ESIGN is based. I conclude this report by summarizing the security of ESIGN Identification in Chapter 6.

Chapter 2

Description of ESIGN Identification

2.1 The setup

ESIGN Identification uses the ESIGN signature scheme. A digital signature scheme has three parts.

Key generation The signer generates a private key and the corresponding public key. He keeps the private key secret and publishes the public key. The authenticity of the public key is certified by a certification authority (CA). The certification authority guarantees with its signature that the verifiers obtain the valid public keys of the signers. It is also possible that the certification authority generates the key pair and gives the secret key to the user.

Signature generation In this step the signer produces the digital signature of a document d . To generate its digital signature, the signer uses his private key.

Signature verification The verifier uses the public key of the signer to verify the digital signature. The signature is convincing if only the signer, knowing his private key, is able to produce that valid signature. The existence

of that valid signature then implies that the signer must have produced it, thereby agreeing to the content of the document.

2.2 ESIGN signature

I give a summary of ESIGN. Technical details can be found in the submission.

2.2.1 Key generation

A security parameter k is fixed which is an integer with $k \geq 320$. Then the following quantities are determined.

1. Two distinct k -bit primes p, q .
2. The *ESIGN modulus* $n = p^2q$.
3. An integer $e > 4$.
4. A hash function $h : \{0, 1\}^* \rightarrow \{0, 1\}^{k-1}$ as described in [2].

The public key is (n, e, k, h) . The private key is (p, q) . I call the integer n an *ESIGN modulus*. The ESIGN description recommends to use $e \geq 8$. In the self evaluation, $e = 32$ is used. I assume that the primes in step 2 are random primes.

2.2.2 Signature generation

I describe the generation of a signature of a document $d \in \{0, 1\}^*$ using the private key from the previous section. The following operations are performed.

1. Choose a random $r \in \{1, \dots, pq - 1\}$ with $\gcd(r, p) = 1$.
2. Set $z = 0||h(d)||0^{2k}$ where $||$ means concatenation.
3. Set $\alpha = (I(z) - r^e) \bmod n$ where $I(z)$ is the integer with binary expansion z .

4. Set $w_0 = \lceil \alpha/pq \rceil$, $w_1 = w_0pq - \alpha$.
5. If $w_1 \geq 2^{2k-1}$ go back to step 1.
6. Set $t = w_0(er^{e-1})^{-1} \bmod p$.
7. Set s to the to length $3k$ normalized binary expansion of $(r+tpq) \bmod n$.
8. The signature is s .

2.2.3 Signature verification

I describe the verification of the signature of the previous section which uses the public ESIGN key from section 2.2. It consists of the following step.

1. If the string consisting of the k most significant bits of the to length $3k$ normalized binary expansion of $I(s)^e \bmod n$ is equal to $0||h(d)$ accept the signature. Otherwise, reject it.

2.3 ESIGN identification

2.3.1 Key generation

The key generation is the same as in the ESIGN signature scheme.

2.3.2 Identification

A *verifier* V wishes to convince himself of the identity of the *prover* P . The secret key of the prover is the private ESIGN key from Section 2.2. The verifier uses the corresponding public ESIGN key. In the identification protocol, the following steps are executed.

1. The verifier V generates a random $r \in \{0,1\}^k$ and sends it to the prover.
2. The prover P computes an ESIGN signature s of r using his private key.

3. The verifier V checks whether the signature is correct using the provers public key. If it is correct, he accepts the identity of the prover P . Otherwise, he rejects it.

Chapter 3

Security proofs

In this chapter I explain what a security proof for a digital signature system is and which security proofs for ESIGN are given in the self evaluation.

3.1 Security of signature schemes

3.1.1 Security proofs are reductions

The security of all known digital signature schemes depends on the intractability of certain computational problems in mathematics, specifically in number theory. Examples are the integer factoring problem and the discrete logarithm problem in an appropriate group. However, no provably hard computational problems are known which can serve as the security basis of a digital signature scheme. Therefore, no rigorous security proofs for signature schemes are known and there is little hope that such proofs will be found in the future.

Today's security proofs are reductions. The goal of such a reduction is to show that the ability of an attacker to mount a successful attack on a signature scheme implies his ability of solving a basic computational problem in mathematics. This is supposed to increase the trust in the security of a digital signature system. The idea of this approach is the following.

When analyzing the security of a signature scheme it is hard to predict which attacks are possible since the system may be very complex and may

depend on numerous parameters. Even, if the underlying basic computational problems are intractable, some part of the signature scheme might be implemented in such a way that an attack is possible. A famous example for such a situation is the discovery of the possibility of an attack against the RSA encryption standard PKCS # 1 (see [3]). In this standard, an insecure padding scheme was used, which compromised the security of the whole scheme, even though the RSA encryption scheme is based on the intractable integer factoring problem. However, if the security of the digital signature scheme can be reduced to the difficulty of a well defined computational problem in mathematics, then, in order to evaluate the security of the digital signature scheme, it is sufficient to study the difficulty of the underlying problem. The difficulty of the underlying mathematical problem can be studied thoroughly and, therefore, the level of security of the signature scheme is easier to estimate.

Such a security reduction also solves another problem. It is possible that a weakness of a digital signature scheme is discovered, for example, by a government agency of some country. That agency may then try to keep this weakness secret and take advantage of it. However, if the weakness of the signature scheme implies that a basic computational problem is no longer intractable, then keeping this weakness secret may be more difficult, since the solution of important scientific problems can be expected to happen at the same time in different places. Hence, reduction proofs make it less likely that a security hole can be abused.

It is an important question what the computational problems are to which the security of digital signature schemes should be reduced in order for the scheme to be considered more secure. Clearly, breaking a digital signature scheme in one of the ways explained below, can be considered to be a computational problem. In this sense, the security of any digital signature scheme can be trivially reduced to the intractability of a computational problem, namely to the problem of breaking itself. However, evaluating the computational difficulty of this problem is very difficult, since it is very complex and has many parameters. This is even more true since breaking a digital signature scheme is a so called *interactive problem*, that is, in that problem several parties are involved: a signer, who knows his secret key, an attacker who does not know that key but wants to generate valid signatures of the signer, and perhaps an honest verifier. In the process of forging signatures the attacker can try to use the help of the signer and the verifier(see Section

3.1.5).

To make the security level of a digital signature scheme easy to evaluate, it is desirable to reduce its security to easy to specify *non-interactive* computational problems. An example is the factoring problem for RSA-modules: Given an integer n which is the product of two large primes p and q , find those factors p and q . It would be optimal to reduce the security of a digital signature scheme to problems which are of mathematical interest independently of their cryptographic applications. Then, the difficulty of those problems would be studied also outside the crypto community and would therefore be easier to evaluate. However, digital signature schemes whose security can be reduced to such problems seem not to be known. In the known reductions, the computational problems depend to a certain extent on the specific digital signature scheme whose security is reduced to them. This is also true in our context and I will discuss this below. In my opinion, the less the computational problems depend on the digital signature schemes the stronger the security proof by reduction is.

3.1.2 Security of the secret key

A minimum requirement for secure digital signature schemes is the security of the secret key. An attacker has access to the public key of the signer. In a secure digital signature scheme, the determination of the secret key from the public key must be infeasible. In the digital signature scheme under review the security of the secret key can be reduced to well studied intractable problems. This will be explained below.

3.1.3 Existential forgery

Suppose that the problem of computing the secret key from the public key is intractable. This does not necessarily mean that the digital signature scheme is secure. It may still be possible that an attacker is able to generate valid signatures without the knowledge of the secret key.

In an *existential forgery* the attacker produces such a signature. In such a forgery, the attacker is not required to have control over the document which is signed. The only requirement is, that the result of an existential forgery is a new signature of some document which has been produced without the

knowledge of the secret key.

3.1.4 No message attacks

A *no message attack* or a *passive attack* is an existential forgery in which the attacker only knows the public key of the signer and has no access to further information such as valid signatures of other documents. A digital signature scheme is considered to be secure against no message attacks, if the possibility of such an attack implies the ability of solving a computational problem which is considered to be intractable.

3.1.5 Adaptive chosen message attacks

I explain the strongest security notion known for digital signature schemes: the security against existential forgery using an *adaptive chosen message attack*.

In an *adaptive chosen message attack* the adversary knows the public signature key of the signer and obtains valid signatures of a sequence of messages of his choice. The messages in the sequence may depend on signatures of previous messages. The goal of the adversary is an *existential forgery*, i.e. he wants to produce a new signature which has not been generated by the legitimate signer. In particular, the signature is not in the sequence of messages whose signatures the attacker has obtained. But the newly signed message is not necessarily a message of the attackers choice.

One practical application of this notion is as follows. Suppose that a signature scheme is used in a challenge response identification, for example in the ESIGN identification scheme. Then the verifier generates challenges which the prover is supposed to sign, thereby proving his identity. Those challenges can be generated as a sequence of adaptive chosen messages. If an adaptive chosen message attack makes existential forgery possible, then the verifier is able to forge valid signatures without knowing the secret key. The use of signature schemes in challenge response identification is quite common.

I explain a method for proving security against chosen message attacks more precisely. In a chosen message attack the attacker can generate a sequence of pairs (message, signature). A message in that sequence may depend

on the previous pairs. The signature generation algorithm is probabilistic. Therefore, the signatures are generated according to some probability distribution. The signature scheme is considered secure against a chosen message attack if it is secure against no message attacks and if without using the secret key it is possible to generate a sequence which is algorithmically indistinguishable (see [1]) from the sequence which is generated using the signature algorithm. The idea of this concept is the following. If an existential forgery is possible using an adaptively chosen sequence of pairs (message, signature), then an existential forgery is possible using the algorithmically indistinguishable simulation of such a sequence. This latter existential forgery is a no message attack since the signing algorithm is not used. However, the digital signature scheme is known to be secure against no message attacks. Therefore, an adaptive chosen message attack is impossible.

It is common belief that security against adaptive chosen message attacks is the strongest possible security notion for digital signature schemes. In other words, no attack against a digital signature scheme is known which cannot be modeled as an adaptive chosen message attack. The role of this security notion is somewhat similar to the role of the model of a Turing machine in the theory of computation. No computing device is known which cannot be modeled as a Turing machine. However, no proof is known that no stronger computing model exists. Likewise, no proof is known that the security against adaptive chosen message attacks is the strongest possible security notion.

In my opinion, if a signature scheme is proven secure against adaptive chosen message attacks, then it can be considered secure in the strongest sense. However, there are no such proofs but only reductions (see Section 3.1.1).

3.1.6 Random oracle model

Security proofs for digital signature schemes are difficult since a digital signature scheme consists of many components and their interaction may be complicated. An important ingredient of most signature schemes are cryptographically secure hash functions. The hash functions map very long messages to short strings of fixed length. The security of hash functions is discussed in Section 4.4. There I explain that no provably secure cryptographic

hash functions are known.

If the security of a signature scheme is analyzed in the random oracle model (see [16], [7]), then the concrete hash function which is used in the digital signature scheme, is replaced by a so called *random oracle*. A random oracle can be viewed as a black box which contains a random function which maps long strings to short strings of fixed length. Nothing is known about this function, but it can be evaluated by making an explicit query. A typical proof of security against passive attacks in the random oracle model works as follows. If it is possible to come up with a forged signature for a document using one random oracle then such a forgery is also possible with another random oracle, resulting in another falsified signature (forking lemma, see [16]). The two valid signatures of the same document can then be used to solve an underlying mathematical problem.

Does a security proof in the random oracle imply the security of the real digital signature scheme in which a concrete hash function is used? Such an implication cannot be proved today. However, assuming that the concrete hash function behaves like a random oracle, a security proof in the random oracle model makes the security of the real scheme more plausible. On the other hand, there exist insecure signature schemes that can be proved secure in the random oracle model (see [7]). Those schemes look fairly artificial. Nevertheless, their existence raises the question what security proofs in the random oracle model really prove.

In my opinion, security proofs in the random oracle cannot prove the security of digital signature schemes but they make their security more plausible.

3.2 ESIGN identification

I first explain that the security of the ESIGN identification scheme can be reduced to the security of the ESIGN digital signature scheme.

The ESIGN identification scheme is a challenge response scheme. In this scheme, a *prover* wants to convince a *verifier* of his identity. The prover has a secret ESIGN key. The verifier obtains the corresponding public ESIGN key from a certification authority (CA). In the identification protocol, the verifier sends a random message to the prover. The prover signs that message. The

verifier verifies the signature. If the signature is correct, then the verifier accepts the identity of the prover. Otherwise he rejects it. If the digital signature scheme and the random number generator used to generate the challenge are cryptographically secure in the sense described in Section 3.1.5, then the challenge response identification scheme is secure. It is, therefore, sufficient to consider the security of the ESIGN digital signature scheme. The random number generator used to generate the challenge will also be examined in this context.

I then following, I discuss the security of the ESIGN signature scheme.

3.3 ESIGN

3.3.1 Security of the secret key

Computing the secret ESIGN key from the public ESIGN key is equivalent to factoring an ESIGN modulus which is $n = p^2q$ where p and q are prime numbers of equal length k , $k \geq 320$.

3.3.2 Security reductions

In the random oracle model, ESIGN is provably secure against adaptive chosen message attacks if the following problem is intractable (see [10]):

AER problem Given a public ESIGN key (n, e, k, h) (see Section 2.2) and a $(k - 1)$ -bit string y . Find $x \in \{0, 1, \dots, n - 1\}$ such that the string which consists of the $k + 1$ leading bits of $x^e \bmod n$ is $0||y$ where $||$ denotes concatenation.

Chapter 4

Basic computational problems

In this chapter I describe the basic computational problems which, given current knowledge, have to be solved in order for ESIGN Identification to be insecure.

4.1 Basics

In this section I explain the terminology which is used in this chapter.

4.1.1 Asymptotic complexity

To estimate the running time and storage requirement of the algorithms that solve the basic problems the function

$$L_x[u, v] = e^{v(\log x)^u (\log \log x)^{1-u}}$$

is used, where x, u, v are positive real numbers. I explain the meaning of this function. We have

$$L_x[0, v] = e^{v(\log x)^0 (\log \log x)^1} = (\log x)^v \tag{4.1}$$

and

$$L_x[1, v] = e^{v(\log x)^1 (\log \log x)^0} = e^{v \log x}. \tag{4.2}$$

Let x be a positive integer which is the input for an algorithm. In the context of this evaluation, x is a positive integer which is to be factored. The binary length of x is $\lfloor \log_2 x \rfloor + 1$.

If an algorithm has running time $L_x[0, v]$, then by (4.1) it is a polynomial time algorithm. Its complexity is bounded by a polynomial in the size of the input. The algorithm is considered efficient, although its real efficiency depends on the degree v of the polynomial.

If the algorithm has running time $L_x[1, v]$, then by (4.2) it is exponential. Its complexity is bounded by an exponential function in the length of the input. The algorithm is considered inefficient.

If the algorithm has running time $L_x[u, v]$ with $0 < u < 1$, then it is *subexponential*. The algorithm is slower than polynomial but faster than exponential. So the function $L_x[u, v]$ can be viewed as a linear interpolation between polynomial time and exponential time.

4.1.2 Practical run times

As usual, the experimental run times of the algorithms are given in *MIPS Years*. One MIPS Year is defined as the amount of computation that can be performed in one year on a single DEC VAX 11/780. Using this terminology, one year of computing on an n -MHz PC is comparable to n MIPS Years. However, this is only a rough estimate of the computing power used, since the computation may have space intensive parts, such as the solution of large linear systems, which cannot be executed on a PC.

4.1.3 Quantum computers

In the early 1980s, Richard Feynman (among others) introduced the idea of a new computing device which is based on the laws of quantum mechanics. Peter Shor [17] was able to prove that on such a quantum computer the integer factoring problem (IFP) and the discrete logarithm problem in finite prime fields have polynomial time solutions. This means that all cryptosystems under consideration here and, more generally, all public-key cryptosystems which are currently being used in practice, are insecure, if quantum computers become practical. There are first experiments with quantum computers,

for example at Los Alamos. However, it is unclear whether quantum computers will ever be practical. For the time being, quantum attacks are not feasible. However, it is necessary to watch the development in the area of quantum computing. Also, it appears to be necessary to develop new digital signature schemes which remain secure even if quantum computers become practical.

4.2 Integer factoring problem

The integer factoring problem (IFP) is the following: Given a positive integer n , find its factorization as a product of prime numbers. Here, the IFP for moduli of the form $n = p^2q$ (ESIGN modulus) with prime numbers p and q of similar binary length is of particular interest.

4.2.1 NFS

The fastest general purpose factoring method is the general number field sieve (NFS) (see [14]).

The running time of NFS is subexponential. More precisely, under plausible assumptions its asymptotic running time can be expected to be

$$L_n[1/3, 1.9229 + o(1)]$$

where the $o(1)$ term goes to zero as n goes to infinity. The storage requirement of NFS is proportional to the square root of the expected running time.

The largest published factorization using the general NFS is that of the 512-bit number RSA 155 which is an RSA modulus of 155 decimal digits, in August of 1999 (cf. [8]). It took less than 10^4 MIPS Years.

Based on this data point, the expected running time of NFS, and assumed algorithmic progress, Lenstra and Verheul [13] predict that IFP is intractable until 2020, if n is at least a 1881-bit number.

If n has the property that there is a polynomial of low degree (4 or 5, in our case) with very small coefficients, which has a zero mod n , then the special number field sieve (SNFS) can be applied. It is much faster than the general number field sieve. Therefore, such moduli must be avoided. It is not

known how to avoid those moduli systematically. But choosing the modulus randomly makes the probability for the SNFS to be applicable negligible.

4.2.2 ECM

The elliptic curve factoring method [15] factors a composite positive integer n in expected time $L_p[1/2, \sqrt{1/2}]$ where p is the smallest prime factor of n . The largest prime factor found so far with ECM was a 54 decimal digit factor in $(6^{43} - 1)^{42} + 1$. Moduli with too small prime factors must be avoided. I expect 100 decimal digit minimal length factors to be secure against ECM attacks for the next 20 years.

4.2.3 Quantum attacks

In [17] Peter Shor shows that IFP can be solved in polynomial time on a quantum computer. It is not yet clear whether quantum computers become ever practical. Currently, quantum attacks do not threaten IFP.

4.3 Random and pseudorandom number generation

The key and signature generation in ESIGN and the challenge generation in ESIGN Identification require the generation of random numbers, specifically random primes. They are in general generated as sequences of random bits.

Such a sequence is generated as follows. A random bit generator (RBG) is used to generate a short sequence of true random bits. Since a RBG is too inefficient, the short true random sequence is expanded by a pseudorandom bit generator (PRBG) into a sequence of the necessary length.

The RBGs used by the scheme under review are not described in the submission. I can therefore not discuss their security here. However, in real applications it is necessary that a cryptographically secure RBG is used.

A PRBG receives as input a random bit sequence and outputs a longer pseudorandom bit sequence. A PRBG is cryptographically secure if an at-

tacker is not able to distinguish its output from a true random sequence in polynomial time.

No provably secure PRBG is known. Several PRBGs are known whose security can be reduced to the intractability of certain number theoretic problems such as the discrete logarithm problem in finite fields (see [6]). However, those PRBGs are not sufficiently efficient.

The PRBG used in practice survive a broad class of statistical tests specified, for example, in [9].

4.4 Hash functions

In signature schemes, hash functions are used to map long documents to short bit strings of a fixed length, which are actually signed. A *collision* of a hash function is a pair of different documents which are mapped to the same hash value. A hash function is called *collision resistant* if finding a collision of that hash function is intractable.

In signature schemes, which sign hash values, the used hash functions must be collision resistant. Otherwise, if a collision (d, d') is found then the two documents d and d' have the same signature. If an attacker is able to obtain a valid signature of d , then he has also a signature for d' . For example, if the signer signs d in a challenge-response authentication, then he has also signed the other document d' , possibly without knowing it. Collision resistant hash functions are one way functions. This means, that computing an inverse image for a given image is intractable. Therefore, in many cases, the use of collision resistant hash functions prevents existential forgeries since even if it is possible to generate a valid signature for a hash value it is impossible to find a document with that hash value.

Using the birthday paradox (see [6]) a collision for a hash function whose image has n elements can be found with probability $> 1/2$ by computing approximately \sqrt{n} hash values. Therefore, the image of the hash function should at least contain 2^{160} elements.

No hash function is known for which the birthday attack is provably the only possible attack. In the past, hash functions such as MD4 have been shown not to be collision resistant. Today, the hash functions SHA-1 [19]

and RipeMD-160 [12] are used in practice. Given current knowledge, they are collision resistant.

The birthday attack can be prevented if a *keyed hash function* is used. This is a function which maps a bit string and a key from a predefined key space to a hash value of fixed length. In the signature process, a random key is generated. The signature algorithm signs the hash value of a document that is generated by the hash function which is parameterized by the chosen key. The key is part of the signature. It is also used in the verification process. In order for digital signature algorithm to be secure, the keyed hash function must be a *universal one-way hash function* (UOWF). This means that given a hash value and a key it is intractable to find a document such that the value of the hash function parameterized by the given key is the given hash value. No provably universal one-way hash function is known. However, there are constructions that use compression functions such as SHA-1 (see [19]) and are assumed to have the universal one-way property.

Chapter 5

Basic cryptographic problems in ESIGN Identification

In this chapter I discuss the hardness of the non-interactive computational problems which are the basis of the security of ESIGN Identification.

5.1 Security of the secret key

In order to find the secret ESIGN key it is necessary to factor the ESIGN modulus n , that is, to solve the following problem.

Problem 5.1.1 (ESIGN factoring problem)

Given positive integers k and n such that $n = p^2q$ where p and q are two k -bit primes. Find p, q .

In the ESIGN description, $k \geq 320$ is suggested. To guarantee the highest possible level of security, the prime numbers p and q should be selected as random primes of length k . This should be added to the description of the system.

Factoring the ESIGN modulus is infeasible if $k \geq 320$. Also, for the ESIGN factoring problem to remain intractable for the next 20 years, Lenstra and Verheul recommend to choose $k \geq 627$.

Although, ESIGN moduli are of a special form, no special purpose attacks against ESIGN are known.

5.2 AER problem

In the random oracle model, the security of ESIGN can be reduced to the AER problem (see Section 3.3.2). The AER problem with $e = 2$ and $e = 3$ was solved in [4] and [5] by lattice basis reduction methods. No such attack is known for $e \geq 4$.

The only known algorithm for solving the AER problem requires the factorization of the ESIGN modulus.

Factoring the ESIGN modulus is infeasible if $k \geq 320$. Also, for the ESIGN factoring problem to remain intractable for the next 20 years, Lenstra and Verheul recommend to choose $k \geq 627$.

Although, ESIGN moduli are of a special form, no special purpose attacks against ESIGN are known.

5.3 Random numbers

No specific random number or random prime generator is recommended. The document refers to the IEEE P1363 standard draft (see [11]). Given the current knowledge, the random number generators recommended there appear to be secure.

5.4 Hash function

The hash function suggested to be used in ESIGN is taken from [2]. This hash function is supposed to solve the padding problem in PKCS #1 (see [3]). However, as is shown in [18] the security proofs in [2] have gaps and appear to be partially incorrect. However, the hash function constructed in [2] seems not to be affected.

Chapter 6

Final evaluation

ESIGN Identification is based on the signature scheme ESIGN. ESIGN Identification is secure if ESIGN is secure.

In the random oracle model (see Section 3.1.6), the security of ESIGN against adaptive chosen message attacks can be reduced to the intractability of the AER problem (see Section 3.3). The AER problem has not been studied before in mathematics. But given current algorithmic knowledge, that problem is intractable. The security reduction in the random oracle model ignores the choice of the hash function. Nevertheless, the random oracle security of ESIGN is a strong argument for its security with the restrictions made in Section 3.1.6 because the choice of the hash function appears to be appropriate.

Given current knowledge, solving the AER-problem (see Section 3.3), which is the security basis of ESIGN, requires factoring the ESIGN modulus. The fastest known general algorithm for factoring an integer n has subexponential complexity $L_n[1/3, 1.9229]$. ESIGN uses moduli $n = p^2q$ where p and q are primes of equal length. However, no factoring algorithm is known which takes advantage of either of those properties. It is my opinion that the AER problem is intractable if the parameters are chosen as suggested in Section 3.3.2.

ESIGN uses a hash function from [2]. Even though, as shown in [18], the security proofs in [2] have gaps and appear to be partially incorrect, the hash function seems not to be affected. Therefore, the hash function in ESIGN appears to be secure.

Not much is said in the ESIGN specification about the generation of random numbers or pseudorandom numbers. The submission refers to the corresponding standards. Therefore, with respect to the random number generation, the proposed scheme appears to be secure.

With the appropriate parameters which are described in this evaluation, ESIGN is a secure digital signature scheme which is of particular interest if provable security reductions are a major concern. Also, ESIGN is a rather simple system. Hence, a correct implementation of ESIGN should be no problem. In the same sense, ESIGN Identification is a secure identification scheme.

I feel that unexpected mathematical breakthroughs in all areas are the most serious threat for ESIGN Identification. It is therefore crucial, that ESIGN is implemented in such a way that it can be easily replaced if it turns out to be insecure.

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